

# Divergences.jl

Divergences is a Julia package that makes it easy to evaluate the value of divergences and their derivatives. These divergences are used to good effects in the package MomentBasedEstimators.

## Definition

A divergence between  $a \in \mathbb{R}^n$  and  $b \in \mathbb{R}^n$  is defined as

$$D(a, b) = \sum_{i=1}^n \gamma(a_i/b_i) b_i,$$

where  $\gamma : D \subseteq \mathbb{R} \rightarrow \mathbb{R}_+$  is convex on twice differentiable on the interior of its domain  $D$ . The divergence function is normalized as to satisfy

- $\gamma(1) = 0$ ,  $\gamma'(1) = 0$ , and  $\gamma''(1) = 0$ .

## Example of divergences

The following divergence types are defined by Divergences.

### Cressie-Read

The type `CressieRead` is a family of divergences. Members of this family are indexed by a function  $\gamma$  indexed by parameter  $\alpha$

$$\gamma_{\alpha}^{CR}(a, b) = \frac{\left(\frac{a}{b}\right)^{1+\alpha} - 1}{\alpha(\alpha + 1)} - \frac{\left(\frac{a}{b}\right) - 1}{\alpha}$$

Notice that  $\nabla_x \gamma_{\alpha}^{CR}$

### Kullback-Leibler divergence

$$\gamma^{KL}(a, b) = \frac{a_i}{b_i} \log(a_i/b_i) - \frac{a_i}{b_i} - 1$$

### Reverse Kullback-Leibler divergence

$$\gamma^{RKL}(a, b) = -\log(a/b) + a/b - 1$$

## Using Divergences

```
using Divergences
```

Suppose  $a = [0.2, 0.4, 0.4]$  and  $b = [0.1, 0.3, 0.6]$ .

```
a = [0.2, 0.4, 0.4]
```

```
b = [0.1, 0.3, 0.6]
```

```
evaluate(KullbackLeibler(), a, b)
```

```
0.0915162218494357
```