1 Quick Review of Vector Calculus

This considers function from R^n into R^m where one or both of n or m is greater than 1.

- functions $f : R \to R$ are called univariate functions.
- functions $f : R^n \to R$ are called scalar functions
- functions $\vec{r} : R \to R^m$ are parameterized curves. The trace of a parameterized curve is a path.
- function $F : R^n \to R^m$, may be called vector fields in applications. They are also used to describe transformations.

When $m > 1$ a function is called *vector valued.*

When $n > 1$ the argument may be given in terms of components, e.g. $f(x, y, z)$; with a point as an argument, $F(p)$; or with a vector as an argument, $F(\vec{a})$. The identification of a point with a vector is done frequently.

1.1 Limits

Limits when $m > 1$ depend on the limits of each component existing.

Limits when $n > 1$ are more complicated. One characterization is a limit at a point c exists if and only if for *every* continuous path going to c the limit along the path for every component exists in the univariate sense.

1.2 Derivatives

The derivative of a univariate function, f, at a point c is defined by a limit:

$$f'(c) = \lim_{h\to 0} \frac{f(c+h) - f(c)}{h},$$

and as a function by considering the mapping c into $f'(c)$. A characterization is it is the value for which

$$|f(c+h) - f(h) - f'(c)h| = \wr(|h|),$$

meaning after dividing the left-hand side by $|h|$ the expression goes to 0 as $|h| \to 0$. This characterization will generalize with the norm replacing the absolute value, as needed.

1.2.1 Parameterized curves

The derivative of a function $\vec{r} : R \to R^m$, $\vec{r}'(t)$, is found by taking the derivative of each component. (The function consisting of just one component is univariate.)

The derivative satisfies