1 Area under a curve

The question of area has long fascinated human culture. As children, we learn early on the formulas for the areas of some geometric figures: a square is b^2, a rectangle $b \cdot h$ a triangle $1/2 \cdot b \cdot h$ and for a circle, πr^2. The area of a rectangle is often the intuitive basis for illustrating multiplication. The area of a triangle has been known for ages. Even complicated expressions, such as Heron's formula which relates the area of a triangle with measurements from its perimeter have been around for 2000 years. The formula for the area of a circle is also quite old. Wikipedia dates it as far back as the Rhind papyrus for 1700 BC, with the approximation of 256/81 for π.

The modern approach to area begins with a non-negative function $f(x)$ over an interval $[a, b]$. The goal is to compute the area under the graph. That is, the area between $f(x)$ and the x-axis between $a \leq x \leq b$.

For some functions, this area can be computed by geometry, for example, here we see the area under $f(x)$ is just 1, as it is a triangle with base 2 and height 1:

```
using CalculusWithJulia   # loads `QuadGK`, `Roots`, ...
using Plots
f(x) = 1 - abs(x)
plot([f, zero], -1, 1)
```

```
Plot{Plots.PlotlyBackend() n=2}
```

Similarly, we know this area is also 1, it being a square:

```
f(x) = 1
plot([f, zero], 0, 1)
```

```
Plot{Plots.PlotlyBackend() n=2}
```

This one, is simply $\pi/2$, it being half a circle of radius 1:

```
f(x) = sqrt(1 - x^2)
plot([f, zero], -1, 1)
```

```
Plot{Plots.PlotlyBackend() n=2}
```

And this area can be broken into a sum of the area of square and the area of a rectangle, or $1 + 1/2$:

```
f(x) = x > 1 ? 2 - x : 1.0
plot([f, zero], 0, 2)
```