Units in NeuronBuilder

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1 Introduction

Two of the papers that most conductance-based models developed in the past decades are based on, Liu et al. (1998) and Prinz et al. (2003), use different units for maximal conductances. Since the user has to provide the *NeuronBuilder* channel constructors with numerical values for these conductances as well as other neuronal parameters, this document describes what units our code accepts and how to convert the values coming from both papers. The choice of units that makes it easy to relate both papers is like in Xolotl (Gorur-Shandilya et al., 2018), where specific conductances are taken per unit area. Before listing all the relevant variables and their units in Table 2, let's review the canon.

2 Conductance-based models of a neuron

All maximal conductances $\bar{g}$ are in Siemens, voltage is in mili Volts. By definition, the relationship between Farads, Amperes, Volts and seconds is $F = \frac{As}{V}$. Remember also that Volts times Siemens gives Amperes: $A = VS$.

2.1 Liu et al. 1998

The voltage equation and the corresponding units are

$$\frac{dV}{dt} = -\sum_i I_i \qquad\qquad \frac{[mV]}{[ms]} = \frac{[nA]}{[nF]} \tag{1}$$

where the ionic currents are

$$I_i = \frac{\bar{g}_i}{C}(V - E_i) \qquad\qquad \frac{[nA]}{[nF]} = \frac{[\mu S]}{[nF]} \cdot [mV] \tag{2}$$

and $C = 0.628\ [nF]$ is the transmembranal capacitance. Maximal conductances are normalized to the surface area of the neuron by dividing the total conductance by the capacitance of the neuron, so their units are $[\mu S/nF]$.

The specific capacitance C_m, which can be (roughly) thought of as a biological constant across many classes of neurons (Gentet et al., 2000), times the surface area of the neuronal compartment, in this case the soma, is the capacitance:

$$C = C_m \cdot Area$$
$$0.628[nF] = 10[nF/mm^2] \cdot 0.0628[mm^2].$$

The somatic compartment is approximated by a cylinder with radius $25\mu m$ and length $400\mu m$ so $Area = 2\pi rl = 0.0628\ [mm^2]$.

The equation for the calcium mechanism that translates calcium current into intracellular calcium concentration, $[Ca]$, with buffering time constant $\tau_{Ca} = 20ms$ is

$$(20ms)\frac{d[Ca]}{dt} = -f \cdot I_{Ca} + [Ca]_\infty - [Ca]; \qquad\qquad \text{with } f_{Liu} = 0.94\frac{[\mu M][nF]}{[nA]}. \tag{3}$$

The factor $f = f_{Liu}$ that multiplies I_{Ca} is given for this particular area, and thus for this particular capacitance $C = 0.628$ $[nF]$.

Summary:

- Conductances $g_{Liu} = \frac{\bar{g}}{C_m \cdot Area}$ incorporate area and specific capacitance *a priori*.

2.2 Prinz et al. 2003

The voltage equation and the corresponding units are

$$C\frac{dV}{dt} = -\sum_i I_i \qquad\qquad [\mu F] \cdot \frac{[mV]}{[ms]} = [\mu A] \tag{4}$$

where the ionic currents are

$$I_i = \bar{g}_i(V - E_i)Area \qquad\qquad [\mu A] = \frac{[mS]}{[cm^2]} \cdot [mV] \cdot [cm^2] \tag{5}$$

and now $C = 0.628 \times 10^{-3}$ $[\mu F]$ has to be in micro-Farads in order to match the scale on the right hand side of 4, namely $[\mu A]$. The area and specific capacitance are considered the same as before, but simply by scaling to centimeters: $Area = 0.628 \times 10^{-3}$ $[cm^2]$ and $C_m = 1[\mu F/cm^2]$.

Finally, the equation for calcium entry is

$$(200ms)\frac{d[Ca]}{dt} = -f \cdot I_{Ca} + [Ca]_\infty - [Ca]; \qquad\qquad \text{with } f_{Prinz} = 14.96\frac{[\mu M]}{[nA]}. \tag{6}$$

To match the micro-Ampere units in the calcium current, the f factor has to be taken as

$$f_{Prinz} = 14.96\frac{[\mu M]}{[nA]} = 14.96 \times 10^3 \frac{[\mu M]}{[\mu A]}.$$

Summary:

- The capacitance is explicitly dividing the right-hand side of the voltage equation but we can re-write equation 4 like $\frac{dV}{dt} = -\sum_i I_i$ if equation 5 is $I_i = \frac{\bar{g}_i Area}{C}(V - E_i)$.
- Conductances $g_{Prinz} = \frac{\bar{g} \cdot Area}{C_m \cdot Area} = \frac{\bar{g}}{C_m}$ incorporate specific capacitance only.

3 Conversion from Liu to Prinz model

The Prinz papers (2003 and 2004) are based on Liu et al. 1998 so equations 4-6 can be recovered from equations 1-3.

Currents in the Liu and Prinz models are

$$I_i = \frac{\bar{g}_i}{C_m \cdot Area}(V - E_i) \qquad \text{and} \qquad I_i = \frac{\bar{g}_i \cdot Area}{C_m \cdot Area}(V - E_i) = \frac{\bar{g}_i}{C_m}(V - E_i)$$

respectively.

Considering the conductances of currents contributing to I_{Ca} in equation 3 as $\bar{g}/C_m$ transforms equation 3 into equation 6. That is, the term $f \cdot I_{Ca}$ in this calcium equation is going to be, with units,

$$\frac{0.94}{0.0628}\frac{[\mu M][nF]}{[mm^2][nA]}\sum_{i \in Ca}\frac{\bar{g}_i}{C_m}(V - E_i)\frac{[nA][mm^2]}{[nF]}.$$

Therefore, rearranging the units on the left-hand side of this last equation (conveniently transferring $[nF/mm^2]$ to the current I_{Ca} - which ends up with pure $[nA]$ units) we see how the two factors f can be recovered from

$$14.96\frac{[\mu M]}{[nA]} = \frac{0.94}{0.0628}\frac{[\mu M]}{[nA]}.$$

There is a modification that does not make the two models correspond completely. The buffering time constant for calcium is 10 times larger in Prinz ($\tau_{Ca} = 200ms$) than it is in Liu. However, $0.94/20 = 0.047$ and $14.96/200 = 0.075$. This means that the calcium dynamics are roughly twice as fast in the Liu model compared to the Prinz model.

4 NeuronBuilder conventions

In *NeuronBuilder* the three equations and their units are

$$\frac{dV}{dt} = \left(\frac{-1}{C_m}\right)\left(\sum_i I_i + I_{app} + I_{syn}\right) \qquad \frac{[mV]}{[ms]} = \left(\frac{[nF]}{[mm^2]}\right)^{-1}\frac{[nA]}{[mm^2]} \tag{7}$$

where the ionic currents are

$$I_i = \bar{g}_i(V - E_i) \qquad \frac{[nA]}{[mm^2]} = \frac{[\mu S]}{[mm^2]} \cdot [mV] \tag{8}$$

and

$$(200ms)\frac{d[Ca]}{dt} = -f \cdot \frac{I_{Ca}}{C_m} \cdot Area + [Ca]_\infty - [Ca]; \qquad \text{with } f = 14.96\frac{[\mu M][nF]}{[nA]}. \tag{9}$$

Note that $\bar{g}_i$ is normalized to unit area. We're actually following Prinz's lead by taking $g_i \mu S/1mm^2$. Also note that the surface area only matters for the calcium equation. Last, but certainly not least, note that the calcium current in the last equation, I_{Ca}/C_m, comes from distributing the $1/C_m$ in the voltage equation.
Equations 7-9 are written as they are coded in Julia, which is a certain way of tracking area and specific capacitance separately.

Like in the documentation for *Xolotl*, the units for our model neurons can be found in this table:

Variable	Value	Units
Area	0.0628	mm^2
Specific capacitance C_m	10	nF/mm^2
Steady-state calcium concentration $[Ca]_\infty$	0.05	μM
Maximal conductances	$0 < \bar{g}$	$\mu S/mm^2$
Synaptic conductances	g_{syn}	$\mu S/mm^2$
Applied current	I_{app}	nA/mm^2

Table 1: Default values for model neuron.

4.1 Conversion from Liu model

NeuronBuilder takes care of the area and specific capacitance by setting them as parameters (albeit constant as the user will usually not change them) in the soma compartment. In order to implement equation 7 exactly as it is formulated here, the only thing that has to be done when getting values of conductances coming from Liu et al. 1998 is to take $1/C_m$ out of them, because it is explicitly handled in the voltage equation. So, multiplying by C_m is all there is to it!

- $g_{Liu} : 1\mu S/nF \rightarrow \times 10nF/mm^2 = 10 g_{Liu}\ \mu S/mm^2 = g_{NB}$

We provide this conversion factor in the code with `L2NB = Cm`.
When building the channels, the scripts will look something like `Liu.Na(700.0 * L2NB)` if the reported value for the sodium conductance in the Liu papers was 700.

4.2 Conversion from Prinz model

Since $g_{Prinz}/Area = g_{Liu}$, and we already know that $C_m \times g_{Liu} = g_{NB}$, then for any value coming from the Prinz papers we must do $(g_{Prinz} \div Area) \times C_m = g_{NB}$.
Once again, users should call the built-in `P2NB = Cm/area` $= 10/0.0628 = 159.2357$. This will make the scripts look something like `Prinz.NaV(100.0 * P2NB)` if the reported value for the sodium conductance in the Prinz papers was 100.

The synaptic conductances given in Prinz et al. 2004 are already scaled to the surface area: their g_{syn} is in fact $\bar{g}_{syn} \cdot Area$ and so, the synaptic current $I_{syn} = g_{syn}(V - E_{syn}) = (\bar{g}_{syn} \cdot Area)(V - E_{syn})$ has the same structure as the ionic currents. Therefore we must divide g_{syn} by the squared-area and scale to micro-Siemens.

- $g_{syn}^{Prinz} : 1nS \rightarrow \times \frac{10^{-3}}{(0.0628)^2 mm^2} = 0.2536\ g_{syn}^{Prinz} \frac{\mu S}{mm^2} = g_{syn}^{NB}$

To get, for example, a glutamatergic synapse with reported synaptic conductance of $30nS$ users should call `convfactor = 1e-3 / (area) ** 2` with `Glut(30.0 * convfactor)`.

Wiktor Phillip's *Conductor* (https://github.com/wsphillips/Conductor.jl) was key to chase down the sources of discrepancy between models.

References

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