

Wilcoxon test via GPC

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Generalized Pairwise comparisons (GPC) include the Wilcoxon rank sum test as a specific case. This vignette explores the connections between the `BuyseTest` output and the more standard implementation of the `wilcoxon-test`.

1 Single Wilcoxon test

1.1 Exact test

Consider the first 'two sample test' example from the help page section of `stats::wilcox.test`:

```
x <- c(0.80, 0.83, 1.89, 1.04, 1.45, 1.38, 1.91, 1.64, 0.73, 1.46)
y <- c(1.15, 0.88, 0.90, 0.74, 1.21)
df <- rbind(data.frame(value = x, group="x"),
            data.frame(value = y, group="y"))
```

We can perform a Wilcoxon test using the `wilcox.test` function:

```
wilcox.test(value ~ group, data = df)
```

```
Wilcoxon rank sum exact test
```

```
data: value by group
W = 35, p-value = 0.2544
alternative hypothesis: true location shift is not equal to 0
```

which, with such a small sample, can perform an exact test, i.e., consider all possible permutation of the group variable. It unfortunately does not output any effect size (just the test statistic and corresponding p-value). The package *asht* contains an alternative implementation:

```
asht::wmwTest(value ~ group, data = df, method = "exact.ce")
```

```
exact Wilcoxon-Man-Whitney test (confidence interval requires
proportional odds assumption, but test does not)
```

```
data: value by group
```

```

Mann-Whitney estimate = 0.3, p-value = 0.2544
alternative hypothesis: two distributions are not equal
95 percent confidence interval:
 0.08292978 0.63269022
sample estimates:
Mann-Whitney estimate
      0.3

```

which output an estimate, the probability that a randomly chosen observation from one group has higher value than a randomly chosen observation from the other group, which is referred to as Mann-Whitney parameter or probabilistic index. To match those results with GPC we can use a permutation test:

⚠ the argument `add.halfNeutral` should be set to `TRUE` to adequately handle ties

```

eperm.BT <- BuyseTest(group ~ cont(value), data = df, add.halfNeutral = TRUE,
                      method.inference = "permutation", n.resampling = 1e4,
                      trace = FALSE, cpus = 5, seed = 10)
confint(eperm.BT, statistic = "favorable")

```

```

      estimate      se lower.ci upper.ci null  p.value
value      0.3 0.1632342      NA      NA 0.5 0.2566743

```

Up to the Monte Carlo error for the p-value calculation, which can be made arbitrarily small by increasing the number of permutations (argument `n.resampling`), the results are identical. Note that the 'default' statistical inference method based on asymptotic U-statistic theory:

```

eU.BT <- BuyseTest(group ~ cont(value), data = df,
                  add.halfNeutral = TRUE)
confint(eU.BT, statistic = "favorable")

```

```

      estimate      se lower.ci upper.ci null  p.value
value      0.3 0.1334166 0.1098282 0.5981833 0.5 0.182315

```

leads to a different p-value as a different null hypothesis is being tested here: probabilistic index equal 0.5 instead of equality in distribution. This p-value corresponds, up to some small sample approximation and the Monte Carlo error, to the one obtain with a studentized permutation:

```

etperm.BT <- BuyseTest(group ~ cont(value), data = df, add.halfNeutral = TRUE,
                      method.inference = "studentized permutation", n.resampling = 1e4,
                      trace = FALSE, seed = 10)
confint(etperm.BT, statistic = "favorable")

```

```

      estimate      se lower.ci upper.ci null  p.value
value      0.3 0.1334166      NA      NA 0.5 0.1916808

```

1.2 Approximate test

Consider now a bigger (artificial) dataset:

```
set.seed(10)
df2 <- rbind(data.frame(value = round(rnorm(50),2), group="x"),
             data.frame(value = round(rnorm(50),2), group="y"))
any(duplicated(df2$value)) ## test whether there are any ties
```

```
[1] TRUE
```

We can again perform a Wilcoxon test using the `wilcox.test` function:

```
wilcox.test(value ~ group, data = df2)
```

```
Wilcoxon rank sum test with continuity correction
```

```
data: value by group
W = 967.5, p-value = 0.05188
alternative hypothesis: true location shift is not equal to 0
```

or, equivalently, with the `wmwTest` function:

```
wmwTest(value ~ group, data = df2)
```

```
Wilcoxon-Mann-Whitney test with continuity correction (confidence
interval requires proportional odds assumption, but test does not)
```

```
data: value by group
Mann-Whitney estimate = 0.613, tie factor = 0.99995, p-value = 0.05188
alternative hypothesis: two distributions are not equal
95 percent confidence interval:
 0.4990803 0.7138973
sample estimates:
Mann-Whitney estimate
 0.613
```

In either case, an exact test would be too computationally demanding and an approximate test is performed instead, which assumes a normally distributed test statistic. The `BuyseTest` package will not be able to match these results due to the continuity correction. Without continuity correction, e.g.:

```
wmwTest(value ~ group, data = df2, correct = FALSE)
```

```
Wilcoxon-Mann-Whitney test (confidence interval requires proportional
odds assumption, but test does not)
```

```

data: value by group
Mann-Whitney estimate = 0.613, tie factor = 0.99995, p-value = 0.05147
alternative hypothesis: two distributions are not equal
95 percent confidence interval:
 0.4992803 0.7137196
sample estimates:
Mann-Whitney estimate
      0.613

```

it is possible to retrieve the exact same p-value by evaluating the variance of the permutation distribution and assuming a normally distributed test statistic. In this simple example this can be done using an analytic formula ([Anderson and Verbeek, 2023](#)):

⚠ the code, kindly provided by the authors of the paper, has been ported to the package with minimal change. It is therefore meant to be used in the context of the original publication and not in the more general setting covered by the package (strata, right-censoring, ...)

```

eperm.BT2 <- BuyseTest(group ~ cont(value), data = df2, add.halfNeutral = TRUE,
                      method.inference = "varexact-permutation")
confint(eperm.BT2, statistic = "favorable")

```

```

      estimate      se lower.ci upper.ci null  p.value
value  0.613 0.05802219      NA      NA 0.5 0.05147115

```

or, more generally, using a resampling method:

```

eperm.BT2 <- BuyseTest(group ~ cont(value), data = df2, add.halfNeutral = TRUE,
                      method.inference = "permutation", n.resampling = 1e4,
                      trace = FALSE, cpus = 5, seed = 10)
confint(eperm.BT2, statistic = "favorable", method.ci.resampling = "gaussian")

```

```

      estimate      se lower.ci upper.ci null  p.value
value  0.613 0.05814569      NA      NA 0.5 0.05118099

```

2 Multiple Wilcoxon tests

Consider now the case where we would like to compare one reference group (here strata a) to multiple treatment groups (here strata b,c,d,e). We will consider the following dataset:

```
set.seed(35)
dt <- simBuyseTest(n.T=25, n.strata = 5)
dt$id <- paste0("id",1:NROW(dt))
dt$strata <- as.character(dt$strata)
print(head(dt), class = FALSE)
```

	id	treatment	eventtime	status	toxicity	score	strata
1:	id1	C	0.03384999	1	yes	0.4777913	b
2:	id2	C	0.65039474	0	no	-1.1048190	d
3:	id3	C	1.00647502	1	no	-0.1407630	b
4:	id4	C	0.01129603	1	yes	-0.5512507	a
5:	id5	C	0.22249748	1	no	1.0465250	d
6:	id6	C	0.07400412	0	no	-2.0053855	d

We can apply the GPC procedure to each pair of group:

```
BuyseTest.options(order.Hprojection=1);BuyseTest.options(trace=0)

ls.BT <- list("b-a=0" = BuyseTest(strata ~ cont(score), add.halfNeutral = TRUE,
                                data = dt[dt$strata %in% c("a","b"),],
                                method.inference = "u-statistic"),
             "c-a=0" = BuyseTest(strata ~ cont(score), add.halfNeutral = TRUE,
                                data = dt[dt$strata %in% c("a","c"),],
                                method.inference = "u-statistic"),
             "d-a=0" = BuyseTest(strata ~ cont(score), add.halfNeutral = TRUE,
                                data = dt[dt$strata %in% c("a","d"),],
                                method.inference = "u-statistic"),
             "e-a=0" = BuyseTest(strata ~ cont(score), add.halfNeutral = TRUE,
                                data = dt[dt$strata %in% c("a","e"),],
                                method.inference = "u-statistic")
)

M.confint <- do.call(rbind,lapply(ls.BT,confint, statistic = "favorable"))
cbind(M.confint,adj.p.value = p.adjust(M.confint[,"p.value"], method = "bonferroni"))
```

	estimate	se	lower.ci	upper.ci	null	p.value	adj.p.value
b-a=0	0.4090909	0.1542200	0.1654639	0.7073759	0.5	0.56434599	1.0000000
c-a=0	0.4375000	0.1465755	0.1948678	0.7142379	0.5	0.67306460	1.0000000
d-a=0	0.2500000	0.1010153	0.1039078	0.4893302	0.5	0.04143057	0.1657223
e-a=0	0.3333333	0.1360828	0.1308601	0.6241219	0.5	0.25767454	1.0000000

Because we compare the treatment groups to the same reference, the test statistics are correlated and a Bonferroni adjustment would not be optimal. A better (but still not optimal adjustment) is the max-test adjustment which can be obtained via the `BuyseMultComp` function:

```
e.mc <- BuyseMultComp(ls.BT, statistic = "favorable", cluster = "id", global = TRUE)
print(e.mc, cols = c("estimate", "se", "p.value", "adj.p.value"))
```

- Multivariate test: p.value = 0.2645493 (df = 4)

- Univariate tests:

	estimate	se	p.value	adj.p.value
b-a=0	0.4090909	0.1542200	0.56434599	0.9289219
c-a=0	0.4375000	0.1465755	0.67306460	0.9752151
d-a=0	0.2500000	0.1010153	0.04143057	0.1223430
e-a=0	0.3333333	0.1360828	0.25767454	0.5831344

Here the smallest p-value has been multiplied by a factor 2.64 instead of 4. This is thanks to the rather strong correlation between the test statistics:

```
M.cor <- cor(lava::iid(e.mc))
dimnames(M.cor) <- list(names(ls.BT), names(ls.BT))
M.cor
```

	b-a=0	c-a=0	d-a=0	e-a=0
b-a=0	1.0000000	0.6519486	0.5601058	0.7520401
c-a=0	0.6519486	1.0000000	0.4240003	0.5439927
d-a=0	0.5601058	0.4240003	1.0000000	0.5051815
e-a=0	0.7520401	0.5439927	0.5051815	1.0000000

References

Anderson, W. N. and Verbeek, J. (2023). Exact permutation and bootstrap distribution of generalized pairwise comparisons statistics. *Mathematics*, 11(6):1502.